

**ORDRE DES INGÉNIEURS DU QUÉBEC
MAY 2026 SESSION**

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Calculators: authorized models only

Exam duration: 3 hours

17-PH-A6 OPTICS

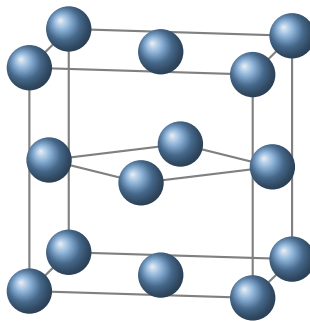
1 Copper Conductivity (10 points)

According to the Drude model, electrons in a metal behave like a gas of free particles that undergo collisions with the ions of the crystal lattice. For copper, the conduction electron density is $8.5 \times 10^{28} \text{ m}^{-3}$ and the mean relaxation time is $\tau = 2.5 \times 10^{-14} \text{ s}$.

- a) (2 points) Give the expression for the electrical conductivity σ according to the Drude model.
- b) (3 points) Calculate the value of the electrical conductivity of this metal.
- c) (3 points) Under the effect of an electric field of $E = 0.1 \text{ V m}^{-1}$, what is the drift velocity of the electrons.
- d) (2 points) Explain why the resistivity of a metal generally increases with temperature.

2 Crystal Structure and Symmetry (10 points)

Consider a crystal with a face-centered cubic (FCC) structure having a lattice parameter $a = 4.05 \text{ \AA}$, like aluminum.



- a) (2 points) Calculate the nearest-neighbor distance and determine the coordination number (how many nearest neighbors does each atom have).
- b) (2 points) How many atoms are there in this FCC unit cell?
- c) (3 points) Calculate the atomic packing fraction, that is, the fraction of volume occupied by atoms assuming they are hard spheres that touch.
- d) (3 points) Calculate the interplanar spacing d_{111} for the (111) planes using the formula for cubic systems.

3 Band Structure in 1D (10 points)

Consider a one-dimensional semiconductor of length L with a parabolic dispersion relation near the conduction band minimum given by:

$$E(k) = E_c + \frac{\hbar^2 k^2}{2m^*}$$

where E_c is the energy at the bottom of the conduction band, m^* is the effective mass, and k is measured with respect to the band minimum.

- a) (3 points) Calculate the group velocity v_g of an electron as a function of k .
- b) (7 points) The density of states $g(E)$ is proportional to $(E - E_c)^\alpha$. Determine α .

4 Phonons (10 points)

The dispersion relation of a one-dimensional monoatomic chain is

$$\omega(q) = 2\sqrt{\frac{K}{M}} \left| \sin\left(\frac{qa}{2}\right) \right|$$

where K is the spring constant, M is the atomic mass and a is the lattice constant.

- a) (2 points) Give the maximum frequency ω_{max} of phonons in this chain.
- b) (5 points) Calculate the speed of sound v_s for this chain.
- c) (3 points) At what temperature T is the thermal energy $k_B T$ equal to the maximum phonon energy? What is this energy called?

A Fundamental Constants

Note: $1.0000 \text{ eV} \leftrightarrow 1.6022 \times 10^{-19} \text{ joule} \leftrightarrow 8065.6 \text{ cm}^{-1}$

Constant	Symbol	Numerical value	Units
Speed of light	c	2.997924583×10^8	$\text{m} \cdot \text{s}^{-1}$
Vacuum permittivity	ϵ_0	8.85×10^{-12}	$\text{F} \cdot \text{m}^{-1} = \text{C}^2 \cdot \text{J}^{-1} \cdot \text{m}^{-1}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}$	$\text{N} \cdot \text{A}^{-2}$
Planck constant	h	6.63×10^{-34}	$\text{J} \cdot \text{s}$
	h	4.14×10^{-15}	$\text{eV} \cdot \text{s}$
	$\hbar$	1.05×10^{-34}	$\text{J} \cdot \text{s}$
	$\hbar$	6.58×10^{-16}	$\text{eV} \cdot \text{s}$
Elementary charge	e	1.6×10^{-19}	C
Bohr magneton	μ_B	927.4×10^{-26}	$\text{J} \cdot \text{T}^{-1}$
	μ_B	5.79×10^{-5}	$\text{eV} \cdot \text{T}^{-1}$
Fine structure	α	7.297×10^{-3}	-
Bohr radius	a_0	0.529	$\text{\AA}$
Electron mass	m_e	9.11×10^{-31}	kg
Proton mass	m_p	1.67×10^{-27}	kg
Avogadro constant	N_A	6.02×10^{23}	mol^{-1}
Boltzmann constant	k	1.38×10^{-23}	$\text{J} \cdot \text{K}^{-1}$
	k	8.62×10^{-5}	$\text{eV} \cdot \text{K}^{-1}$
Atomic mass unit	amu	1.66×10^{-27}	kg